H-2325

First Year M. A. Examination

May/June – 2018

Mathematics : Paper-406

(P.D.E. & Fourier Analysis)

Time : 3 Hours] [Total Marks : 100

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **FIRST YEAR M. A.**

Name of the Subject :

☛ **MATHEMATICS : PAPER-406**

☛ Subject Code No. : **2 3 2 5** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) There are five questions in the question paper.

(3) Answer all questions.

(4) Figure to the right indicates marks of the questions.

Q.1

(a) Determine Simultaneous differential equations for the first order and the first degree in three variables. 7

(b) Define: Pfaffian differential equation. Show that a Pfaffian differential equation in two variables always possesses an integrating factor. 7

(c) Solve the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ 6

OR

(a) If X is a vector space such that $X.\,curl\,X = 0$ and μ is an arbitrary function of x, y, z then prove that $\mu X.\,curl\,\mu X = 0$ 7

(b) show that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and also find its primitive. 7

(c) State and prove Natani's method. 6

Q.2

(a) Solve the equation $(y + z)dx + (z + x)dy + (x + y)dz = 0$ 7

(b) Find the equation for integral surfaces passing through any given curve 7

(c) Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. 6

OR

(a) State and prove Charpit's method. 7

(b) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$. 7

(c) If a characteristic strip contains at least one integral element of $F(x, y, z, p, q) = 0$ then show that it is an integral strip of the equation $F(x, y, z, z_x, z_y) = 0$. 6

Q.3

(a) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis. 7

(b) Find a complete integral of the equation $p^2 y(1 + x^2) = qx^2$ 7

(c) Describe Jacobi's method. 6

OR

(a) If $u_1, u_2, \ldots, u_n$ are solution of the homogenous linear partial differential equation $F(D, D')z = 0$ then prove that $\sum_{r=1}^{n} c_r u_r$; where the $c_r's$ are arbitrary constants, is also a solution. 7

(b) Find the particular integral of the equation $(D^2 - D')z = e^{2x+y}$. 7

(c) Solve the equations $r + s - 2t = e^{x+y}$. 6

Q.4

(a) Find the Fourier series for the function $f(x) = \left(\frac{\pi - x}{2}\right)^2 ; 0 < x < 2\pi$ 7

(b) Expand the function $f(x) = e^x, -\pi < x < \pi$ in terms of Fourier series. 7

(c) Derive the complex Fourier series for the interval $[0, 2\pi]$. 6

OR

(a) Define orthonormal system of functions. Prove that the sum of the squares of the fourier co-efficient of a square integrable function always converges. 7

(b) Find the Fourier series to represent the function $f(x) = x^2,\ for\ x \in [-\pi, \pi]$ and use Parseval's identity to show $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} \ldots = \frac{\pi^4}{90}$ 7

(c) Define odd and even functions. Discuss about half range sine and cosine fourier series. 6

Q.5

(a) Prove the followings: 7

1) $\frac{1}{\pi}\int_{-\pi}^{\pi} D_n(u)du = 1$; $where\ D_n(u) is\ Dinchlet's\ kernel$

2) $D_n(u) = \frac{\sin(n+\frac{1}{2})u}{2\sin(\frac{u}{2})}$

(b) Define orthogonal system and complete orthonormal system of functions. Prove that if $\{\emptyset_n(x)\}$ be a complete orthonormal family of functions and $f\ and\ g$ be integrable over $[a, b]$ then $f = g$ iff fourier expansion of f is equal to the fourier expansion of g 7

(c) Derive Fourier integral formula. 6

OR

(a) Find the cosine transform of x defined as: $f(x) = \begin{cases} 1 \ ; \ 0 \leq x < a \\ 0 \ ; \ x \geq a \end{cases}$. What is the function whose cosine transform is $\sqrt{\frac{2}{\pi}}\left(\frac{\sin ak}{k}\right)$? 7

(b) Show that 7

(a) $\mathcal{F}_c\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\left(\frac{a}{a^2+k^2}\right) \ ; \ a > 0$

(b) $\mathcal{F}_s\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\left(\frac{k}{a^2+k^2}\right) \ ; \ a > 0$

(c) Prove that the sum of the squares of the Fourier co-efficient of a square integrable function always converges. 6
